International Journal of Theoretical Physics, Vol. 45, No. 4, April 2006 (© 2006)
DOI: 10.1007/s10773-006-9060-0

Model-Theoretic Investigations into Consequence Operation (Cn) in Quantum Logics: An Algebraic Approach

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Received July 21, 2005; accepted January 26, 2006
Published Online May 23, 2006

In this paper, we present the fundamentals of the so-called algebraic approach to propositional quantum logics. We define the set of formulae describing quantum reality as a free algebra freely generated by the set of quantum proportional variables. We define the general notion of logic as a structural consequence operation. Next, we introduce the concept of logical matrices understood as a model of quantum logics. We give the definitions of two quantum consequence operations defined in these models.

KEY WORDS: algebraic logic; consequence operation; logical matrices; models of qunatum logics.

1. INTRODUCTION

Historically speaking, we can distinguish two different and competitive ways of understanding of the concept of "logic." An approach considering the logic as a set of logically valid sentences was the first manner of understanding logic. In this approach, one can perceive a logical system as a set of sentences closed under substitutions and some rules of inference. A paradigmatic example is a set of tautologies of classical propositional calculus. Second and more general approach enables one to comprehend a logic as a logical consequence operation (or relation). This approach formalizes the most general principles of reasoning and not a set of logically valid sentences. Following the second approach, one will uniquely obtain a set of logically valid sentences as a set of consequences of an empty set of premises. Following the first approach, i.e., starting from a set of logically valid sentences, one will not obtain a uniquely determined consequence operation. So, there usually exist plenty of consequence operations for a given logical system.

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Summing up above considerations can claim that logical validity does not determine the rules of reasoning. Hence, the notion of logic can be understood as a structural consequence operation discussed in detail in Section 3. In the literature concerning quantum logic, there are only several articles dealing with quantum logic as a structural consequence operation. In the opinion of many logicians the notion of logic as a structural consequence operation is one of the most important logical concepts. Considering logic as a structural consequence, operation belongs to the heritage of the Lvov–Warsaw school of logic and constitutes the basis for the development of so-called abstract algebraic logic (Font *et al.*, 2003). The process of logical system algebraization is rooted in the belief that this process allows us to investigate general properties of logical systems.

The idea of a logical calculus based on the relation between the properties of a physical system and the self-adjoint projection operators defined on a Hilbert space can be traced back to the work of von Neumann (Birkhoff and von Neumann, 1936). In our papers, we follow the so-called Geneva Approach to the foundations of quantum physics (Aerts, 1999; Piron, 1976). This approach can be alternatively termed Operational Quantum Logic (Smets, 2001) and corresponds to the theory of "Property Lattices."

The general idea of Operational Quantum Logic is to give a complete formal description of physical systems in terms of their actual and potential properties and a dual description in terms of their states.

Fundamental notion of quantum logic is that of "yes–no question" or "definite experimental project". A "yes–no" question $\alpha \in Q$ is an experimental procedure and can be understood as a list of concrete actions accompanied by a rule that specifies in advance with outcomes count a positive response. A question is named "true" for a particular physical system if it is certain that "yes" would be obtained when the experimental procedure is performed, and is called "not true" otherwise (Smets, 2001).

The main point being that the structure of mathematical representatives for experimental propositions of a quantum system, corresponding to the projections on a Hilbert space forms an orthomodular lattice—or equivalently—can be modeled by orthomodular lattices.

Quantum logics (just like classical logic) are a kind of propositional logic. They are determined by a class of algebras. These algebras are defined by a set of identities. In other words, each logic is formalized by a set of axiom schemes and inference rules which correspond to its defining set of identities. These logics represent a natural logical abstraction from the class of all Hilbert space lattices. They are represented respectively by orthomodular quantum logic (OQL) and by the weaker orthologic (OL), which for a long time has been also termed minimal quantum logic.

This article tries to define two different notions of quantum consequence operations: the weak one and the strong one (Section 3). In order to do that, we must define the quantum sentential calculus as an absolutely free algebra (Section 2). We will give full model-theoretic characterization of quantum logic, which enables us to define two quantum consequence operations (Section 4).

2. PRELIMINARY REMARKS

Every algebra we consider here has the signature $\langle A, \leq, \cap, \cup, (.)', 0, 1\rangle$ and is of similarity type $\langle 2, 2, 1, 0, 0\rangle$. Algebraic structures, in particular algebras, will be labeled with set of boldface complexes of letters beginning with a capitalized Latin characters, e.g., **A**, **B**, **Fm**, . . ., and their universes by the corresponding light-face characters, A, B, Fm, ... All our classes of algebra are varieties (we define variety as an equationally definable class of algebra). The varieties of ortholattices are denoted by **OL**. In order to show that, this class constitutes a variety explicitly, we give its definition by the set of identities:

Definition 1. An Ortholattice is an algebraic structure $\mathbf{U} = \langle A, \leq, \cap, \cup, ', 0, 1\rangle$ which satisfies the following identities:

$$x \cap y = y \cap x$$
$$x \cap (y \cap z) = (x \cap y) \cap z$$
$$x = x \cap (x \cup y)$$

$$x \cup y = y \cup x$$
$$x \cup (y \cup z) = (x \cup y) \cup z$$
$$x = x \cup (x \cap y)$$

$$x \cup 1 = 1$$
$$x \cap x' = o$$
$$(x')' \cap x = x$$
$$x' \cap (x \cup y)' = (x \cup y)'$$
$$x'' = x$$

In other words, an ortholattice is a bounded lattice with a unary operation $(.)'$ which satisfies the following: for any $x, y \in A$

(a) $x \leq x''$
(b) $x \cap x' = 0$
(c) $x \leq y$ implies $y' \leq x'$

The variety of **OML** of all orthomodular lattices, the class **MOL** of all modular ortholattices and the class **BA** of all Boolean algebras are defined by adding the orthomodular law, the modular law and the distributive law respectively, to the identities for **OL**.

One can represent it as follows:

For **OML** $x \cap \{(x \cap y) \cup x'\} = x \cap y$ (orthomodular law)

For **MOL** $x \cap \{(x \cap y) \cup z\} = (x \cap y) \cup (x \cap z)$ (modular law)

For **BA** $x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$ (distributive law)

All classes, we mention here are varieties being subvarieties of **OL**, and the relation between these varieties is:

$$\mathbf{BA} \subseteq \mathbf{MOL} \subseteq \mathbf{OML} \subseteq \mathbf{OL}$$

Of course, there are many other subvarieties of **OL** not mentioned here.

In this introductory exposition we adopt a framework of binary logic introduced by Goldblatt (1974). First, we define the system for a binary logic, which corresponds to the **OL** variety, and then we extend this system by introducing several axiom schemes.

Definition 2. An orthologic OL on the set of formulae includes the following axioms and is closed under the following inference rules:

Axiom schemes	Inference rules
(Ax 1) $\alpha \vdash \alpha$	(R 1) $\frac{\alpha \vdash \beta \beta \vdash \gamma}{\alpha \vdash \gamma}$
(Ax 2) $\alpha \vdash \neg\neg\alpha$	
(Ax 3) $\alpha \wedge \beta \vdash \alpha$	(R 2) $\frac{\alpha \vdash \beta \alpha \vdash \gamma}{\alpha \vdash \beta \wedge \gamma}$
(Ax 4) $\alpha \wedge \beta \vdash \beta$	
(Ax 5) $\alpha \vdash \alpha \vee \beta$	(R 3) $\frac{\alpha \vdash \gamma \beta \vdash \gamma}{\alpha \vee \beta \vdash \gamma}$
(Ax 6) $\beta \vdash \alpha \vee \beta$	
(Ax 7) $\alpha \wedge \neg\alpha \vdash \beta$	(R 4) $\frac{\alpha \vdash \beta}{\neg\beta \vdash \neg\alpha}$
(Ax 8) $\neg\neg\alpha \vdash \alpha$	

Subsequent logics are defined by adding additional axiom schemes:

The orthomodular logic (OML)	$\alpha \wedge (\neg\alpha \vee (\alpha \wedge \beta)) \vdash \beta$
The modular orthologic (MOL)	$\alpha \wedge ((\alpha \wedge \beta) \vee \gamma) \vdash (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$
The classical logic (CL)	$\alpha \wedge (\beta \vee \gamma) \vdash (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$

The relation between strengths of these logics is shown below:

$$\text{OL} \rightarrow \text{OML} \rightarrow \text{MOL} \rightarrow \text{CL} \rightarrow \text{inconsistent logic}$$

In considering propositional quantum logic, we follow the path taken by algebraically oriented logicians. We define a sentential language as an absolutely free algebra. As a consequence of such definition we can adequately describe basic properties of the propositional language (Font *et al*., 2003).

First, we introduce the notion of the algebra of formulae and we denote it by **Fm**. **Fm** is absolutely free algebra of type L over a denumerable set of generators Var $= \{p, q, \ldots, r\}$. The set of generators—Var—is identified with the countable infinite set of propositional variables. The universe of **Fm** algebra is formed of inductively defined formulae. The set of formulae describing quantum entity is inductively defined as the least set satisfying the following conditions:

(1) Var $\subset$ Fm where Var $= \{p, q, \ldots, r\}$ is the set of quantum propositional variables
(2) If $p, q, \ldots, r \in$ Fm, then finite sequence $F_i$pqr also belongs to **Fm** for any $i = 1, 2, \ldots, n$.

The **Fm** algebra is endowed with finitely many finitary operations (connectives) $F_1, F_2, \ldots, F_n$. Thus, **Fm** consists in the set of formulae together with the operations of forming complex formulae associated with each connective. The structure **Fm** $= \langle$Fm, $F_1, F_2, \ldots, F_n\rangle$ is called the algebra of formulae—or equivalently—the algebra of terms. The similarity type L of the algebra depends on the number and arity of connectives.

The definition of language as a free algebra allows us to treat sentential connectives as algebraic operations. The process of formation of complex propositions from atomic ones is the algebraic process occurring between elements of a given algebra.

3. CONSEQUENCE OPERATION AND LOGICS

In 1930, Tarski (1983) defined what later on was called finitary consequence operation – Cn. A consequence operation is a particular case of a closure operation (Burris and Shankapanavar, 1981). Consequence operation is a structural consequence operation defined on the algebra of formulae if Cn satisfies the following conditions (Font *et al.*, 2003; Tarski, 1983):

(i) $X \subseteq \mathrm{Cn}(X)$ reflexivity
(ii) if $X \subseteq Y$ then $\mathrm{Cn}(X) \subseteq \mathrm{Cn}(Y)$ monotonicity
(iii) $\mathrm{Cn}\,\mathrm{Cn}(X) \subseteq \mathrm{Cn}(X)$ idempotency
(iv) $e\mathrm{Cn}(X) \subseteq \mathrm{Cn}(e(X))$ structurality

The last condition says that Cn is closed with respect to substitution i.e., Cn is substitution-invariant. Algebraically speaking, substitutions occurring in the algebra of terms can be understood as an endomorphism of these formulae.

Substitutions in the sentential language are defined as functions from a set of sentential variables into the set of formulae. Formally, a substitution is the function

$$\mathrm{e}:\ \mathrm{Var} \rightarrow \mathrm{Fm}$$

Based on the fact, that the algebra of terms is the free algebra the function e can be extended to an endomorphism

$$h^e : \mathrm{Fm} \to \mathrm{Fm}$$

(v) Additionally, if Cn satisfies the following condition:

$$\mathrm{Cn}(X) = \bigcup \{\mathrm{Cn}(Y) : Y \subseteq X, Y \text{ is finite}\}$$

it is called a finitary consequence operation.

A consequence operation Cn on a set of formulae can be easily transformed into a consequence relation $\vdash_{\mathrm{Cn}} \subseteq P(\mathrm{Fm}) \times \mathrm{Fm}$ between subsets of Fm and elements of Fm by postulating for every $X \subseteq \mathrm{Fm}$ and every $\alpha \in \mathrm{Fm}$ that

$$X \vdash_{\mathrm{Cn}} \alpha \quad \text{if and only if} \quad \alpha \in \mathrm{Cn}(X)$$

P (Fm) is a power set of Fm.

A consequence relation inherits all its properties from properties of consequence operation (i–v).

In our algebraic approach we identify the general notion of logic with the structural consequence operation. The logic or deductive system in the language of type L is a pair $S = \langle \mathbf{Fm}, \vdash_S \rangle$ where **Fm** is the algebra of formulae of type L and $\vdash_S$ is a substitution-invariant consequence relation on Fm, that is, a relation $\vdash_S \subseteq P(\mathrm{Fm}) \times \mathrm{Fm}$ satisfying the conditions (i–iv). A logic S is said to be finitary when its consequence relation satisfies the relational form of property (v), that is, when for every $\Gamma \cup \{\phi\} \subseteq \mathrm{Fm}$:

$$\text{If } \Gamma \vdash_S \phi \quad \text{then there is a finite} \quad \Gamma' \subseteq \Gamma \quad \text{such that } \Gamma' \vdash_S \phi$$

In our article we restrict ourselves only to finitary logics.

An identification of the notion of logic with the notion of structural consequence operation points out in one-to-one correspondence the set of all theories, which can be defined on the set of formulae. The sets of the form $X = \mathrm{Cn}(X)$ are called theories or deductive systems. On a fixed set of formula—Fm—one can define many different structural consequence operations. The set of all structural consequence operations form a complete lattice.

Based on Dishkant's work (1974), we treat the language of quantum logics as a free algebra. In the literature dealing with quantum logics, there exist two different notions of logical consequence. They are determined by a class of orthomodular lattices. The first introduced notion of logical consequence in quantum logic is created by Kalmbach (1983). A sentence α is a weak logical consequence of the set X of sentences if and only if in every model and every valuation in which, every sentence of the set X has a unit of certain orthomodular lattice as its logical value, the sentence α has the unit as its logical value, too.

In 1974, Goldblatt introduced the notion of strong quantum logical consequence: sentence α is a strong logical consequence of the set of sentences X if and only if for any orthomodular lattice A from a given orthomodular lattices and any valuation v, $v(\beta) \leq v(\alpha)$ for every $\beta \in X$ (the symbol $\leq$ denotes the lattice order of A).

All above concepts of quantum logical consequence presuppose the notion of the model of quantum logics.

4. MODELS OF QUANTUM LOGICS

In our investigation, we employ the general method of constructing the models of sentential calculus. We use the so-called matrix method, which allows us to give a full algebraic description of quantum logics (Wójcicki, 1973, 1988).

By a logical matrix we mean a couple $M = \langle \mathbf{A}, F \rangle$ where **A** is an algebra of the same similarity type as the algebra of terms of considered sentential language and F is a subset of A called the set of designated elements of M. As indicated we rule out neither that the set of designated elements $F = \emptyset$ nor that $F = A$. The matrices of the form $\langle \mathbf{A}, \emptyset \rangle$ and $\langle \mathbf{A}, \mathrm{A} \rangle$ are referred to as trivial.

The general concept underlying the notion of logical matrix is that the algebra of matrix A is similar to the algebra of formulae of a given propositional language. In our case, the algebra A is similar to the algebra of terms of quantum logics in the sense of Dishkant (1974). Such logical matrix can be understood as an algebraic semantical model of the considered language or simply as algebraic semantics for quantum logics.

The set **A** can be considered as a range of variability of propositional variables. This set can be regarded as a set of semantic correlates of sentential variables (or alternatively as a set of algebraic correlates of sentential variables; Wójcicki, 1973, 1988).

The concept of logical matrices regarded as models for sentential logics is of particular importance. Every logical matrix consists of an algebra, which is homomorphic with the algebra of terms of a given sentential language. Logical matrices associated with quantum logics are formed of a variety of **OL** or **OML**. These are "natural" classes of homomorphic algebras forming logical matrices. There are many open questions as to whether other algebras e.g., Jordan algebras or Grassmann algebra, can form logical matrices for the algebra of terms of quantum propositions.

The above hints can be understood as a link between purely logical considerations concerning bases of quantum theory and mathematical investigations aiming at finding an appropriate algebraic structures describing quantum reality. In this paper we restrict ourselves only to "natural" algebraic semantics for quantum logics, i.e., the variety of **OL** and **OML**.

Each formula ϕ of the language of quantum logic has a unique interpretation in **A** depending on the value in **A** that are assigned to its variables.

Based on the facts that **Fm** is absolutely freely generated by a set of variables (the set of free generators) and that **A** is an algebra of the same similarity type as **Fm**, there exist a function *f*: Var $\rightarrow$ A and exactly one function h^f: Fm $\rightarrow$ A, which is the extension of the function *f* i.e., $h^f(p) = f(p)$ for each $p \in$ Var. This function is the homomorphism from the algebra of formulae into the algebra **A** of the logical matrix $M = \langle \mathbf{A}, F \rangle$.

Now we can identify the interpretation of a given formula ϕ of **Fm** with h (ϕ) where h is a homomorphism from **Fm** to **A** that maps each variable of ϕ into its assigned value. A homomorphism whose domain is the algebra of terms is called an assignment. One can alternatively write a formula ϕ in the form $\phi(x_0, \ldots, x_{n-1})$ to indicate that each of its variables occurs in the list $x_0, \ldots, x_{n-1}$ and we write $\phi^{\mathbf{A}}(a_0, \ldots, a_{n-1})$ for h (ϕ) where h is any assignment such that h $(x_i) = a_i$ for all $i < \omega$. Given a quantum logic S in a language of type L, an L-matrix $\langle \mathbf{A}, F \rangle$ is said to be a model of S if for every $h \in$ Hom and every $\Gamma \cup \{\phi\} \subseteq$ Fm

$$\text{if } h\ [\Gamma] \subseteq F \text{ and } \Gamma \vdash_S \phi \text{ then } h(\phi) \in F$$

In this case it is also said that F is a deductive filter of S or, as is common now, an S-filter of **A** (Font *et al.*, 2003; Wójcicki, 1988). Given an algebra **A** of similarity type L, the set of all S-filters of **A**, which is denoted by $Fi_S\mathbf{A}$ is closed under intersection of an arbitrary family and is thus a complete lattice (Font *et al.*, 2003). By $h \in$ Hom(**Fm**, **A**) we mean an homomorphism from the algebra of terms into the algebra forming the logical matrices for quantum logics.

Given any set of formulae $X \subseteq A$, there is always the least S-filter of **A** that contains X. It is called the S-filter of **A** generated by X and is denoted by $Fi_S^A(X)$.The class of all matrix models of quantum logic S is denoted by **Mod**S or **K.**

Every logical matrix points out to a set of its own tautologies i.e., a set of formulae such that h $(\alpha) \in F$ for $\alpha \in$ Fm for every homomorphisms $h \in$ Hom(**Fm**, **A**). The set of all tautologies of given matrices is denoted by E (M). It is invariant with respect to the endomorphisms of the algebra of terms. Every invariant set of formulae $X \subseteq$ Fm may be represented as E $(M) = X$ with an appropriate matrix M. The above is the well known as Lindenbaum's theorem (Los and Suszko, 1962). For the purpose of its proof it is enough to consider the matrix of the form $\langle \mathbf{Fm}, X \rangle$. The matrices of this form are termed Lindebaum's matrices. For such matrices the valuations are simply endomorphisms of **Fm** (Los and Suszko, 1962).

Also every logical matrix determines a so-called matrix consequence operation—C_M. For arbitrary $X \subseteq$ Fm

$$C_M(X) = \bigcap \{h^{-1}(F) : h(X) \subseteq F, h \in \text{Hom}(\mathbf{Fm}, \mathbf{A})\}$$

or equivalently: For arbitrary $X \subseteq \mathrm{Fm}$ and for arbitrary formula $\alpha \in \mathrm{Fm} : \alpha \in C_M(X) \leftrightarrow$ for every h $\in$ Hom(Fm, A)$if\, h(\beta) \in F$ for every $\beta \in X$ then $h\,(\alpha) \in F$.

For every matrix the operation defined in such a manner is a structural and uniform consequence. We call it the matrix-consequence (C_M; Los and Suszko, 1962).

In opinion of many logicians, the above statements present the nearest connection between sentential logics and interpretations by matrices (Los and Suszko, 1962).

We ask what is the relationship between structural consequence operation defined in Section 3, particularly strong and weak quantum logical consequence and the so-called matrix consequence. We present here the theorem [without proof, see Los and Suszko, 1962; Wójcicki, 1988] establishing the conditions, which must be satisfied in order to $\mathrm{Cn} = C_M$.

Theorem (Los–Suszko, 1958; Wójcicki, 1988) *Let Cn be structural consequence operation (logic). Then Cn is a matrix consequence if and only if Cn is absolutely uniform.*

We call a consequence Cn *uniform* if and only if for all set of formulae $X, Y \subseteq \mathrm{Fm}$ and for a formula $\alpha \in \mathrm{Fm}$, the following conditions are satisfied:

i) Var $(X, \alpha) \cap$ Var $(Y) = \emptyset$
ii) Var $(Y) \neq \mathrm{Fm}$, Fm being the set of all formulae
iii) $\alpha \in \mathrm{Cn}(X \cup Y)$ then
iv) $\alpha \in \mathrm{Cn}(X)$

The symbol Var (X) means all free sentential variables of the set of formulae X. The equation Var $(X, \alpha) \cap \mathrm{Var}(Y) = \emptyset$ means that the set $(X \cup \{\alpha\})$ and Y have no variables in common.

The logic Cn is said to be *separable* if and only if given two sets of formulae X, Y of the language of Cn such that Var $(X) \cap Var(Y) = \emptyset$ and given any variable $r \notin \mathrm{Var}(X \cup Y)$ the following condition is satisfied:

$$\text{If } r \in \mathrm{Cn}(X \cup Y) \text{ then either } r \in \mathrm{Cn}(X) \text{ or} \mathrm{Cn}(Y)$$

The separability condition can take the following stronger form.

A consequence Cn will be said to be absolutely separable if and only if for each family R of sets of formulae such that for any two sets $X, Y \in R$ if $X \neq Y$ then Var $(X) \cap \mathrm{Var}(Y) = \emptyset$ and for each propositional variable $r \notin \mathrm{Var}(\cup R)$

$$\text{If } r \in \mathrm{Cn}\left(\bigcup R\right) \text{ then } r \in \mathrm{Cn}(X) \text{ for some } X \in R$$

A consequence that is both *uniform* and *absolutely separable* will be called *absolutely uniform*.

The logical matrices determining consequence operation, which is equal to the structural consequence operation i.e., $\mathrm{Cn} = C_M$ are called strongly adequate logical matrices (Wójcicki, 1988).

As it is stated in Section 3 in the language of quantum logic, we can define two different consequence operations: the weak one and the strong one.

The strong consequence operation is determined by the class of models of quantum logic as follows:

$$\Gamma \vdash \phi \text{ iff } \forall \mathbf{A} \in \mathbf{OML}, \forall h \in \mathrm{Hom}(\mathbf{Fm}, \mathbf{A}) \forall a \in A$$
$$\text{if} \quad a \leq h(\beta) \forall \beta \in \Gamma \quad \text{then} \quad a \leq h(\phi)$$

The weak consequence operation is determined by the class of models of quantum logic as follows:

$$\Gamma \vdash \phi \text{ iff } \forall \mathbf{A} \in O\mathbf{ML}, \forall h \in \mathrm{Hom}(\mathbf{Fm}, \mathbf{A}) \text{ if } h(\beta) = 1 \forall \beta \in \Gamma \quad \text{then } h(\phi) = 1$$

The names "weak" and "strong" are misleading because the weak quantum consequence operation is the strengthening of the strong quantum consequence operation (Malinowski, 1992).

In the above formal exposition of the two different definitions of quantum logical consequence, we consider an algebra **A** as belonging to the variety of **OML**. Based on the definition of quantum logical consequence we can uniquely point out the classes of algebras constituting the matrix (algebraic) semantics for quantum logics.

Corollary 1. *The class of matrices*

$$\mathbf{Mod}\mathrm{S} = \{(\mathbf{A}, [a) : \mathbf{A} \in \mathbf{Mod}\mathrm{S}, a \in A\}$$

is a matrix semantics for the strong version of quantum logic. $[a)$ *is a principal filter of the form* $\{x \in A : x \geq a\}$

Corollary 2. *The class of matrices*

$$\mathbf{Mod}\mathrm{S} = \{(\boldsymbol{A}, \{1\}) : A \in \mathbf{Mod}\mathrm{S}\}$$

is a matrix semantics for the weak version of quantum logic.

5. CONCLUSION

In our paper, we did not consider any physical implications of different forms of quantum logical consequence operations. Following the main idea that any logic can be understood as a structural consequence operation, we indicated adequate semantics for quantum logics. Investigations carried out in this paper consist first report concerning more general topic—"Inference in Quantum Logics." We plan to present consequence operation define on Greechie diagram. In order to do that, we will introduce the notion of Greechie diagram satisfiability. Results of these investigations will be presented elsewhere.

These are also reports treating consequence operation in quantum logics as a kind of nonmonotonic reasoning (Engesser and Gabbay, 2002). Above approach will be confronted with our statements considering consequence operation in quantum logics as a kind of monotonic reasoning.

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